Module 5

Fluid Mechanics

Session Slides with Notes

Fluid Mechanics

Density

SI units - $\frac{kg}{m^3}$

$$\rho = \frac{m}{V}$$

ρ = density
m = mass
V = volume

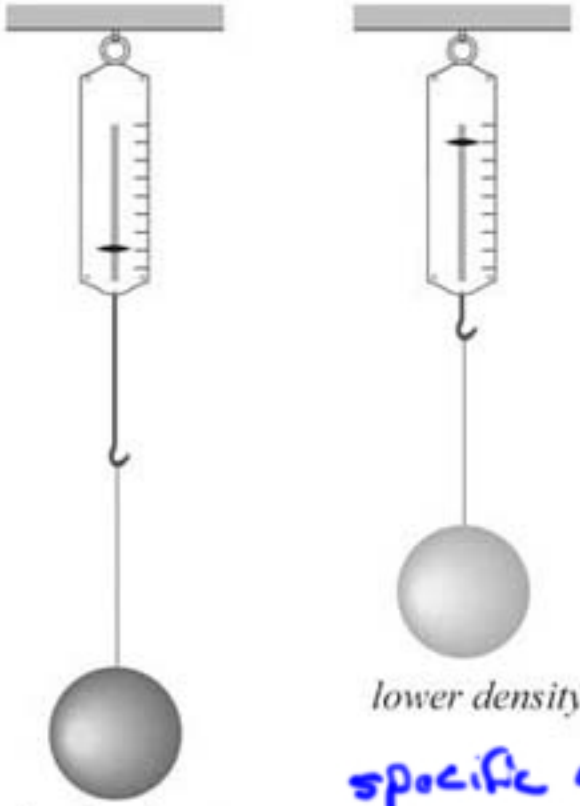
$\rho_{H_2O} = \frac{1000 kg}{m^3}$

$= \frac{1 g}{cm^3} = \frac{1 g}{cc}$

$= \frac{1 g}{mL}$

$= \frac{1 kg}{L}$

Density is the mass per unit volume



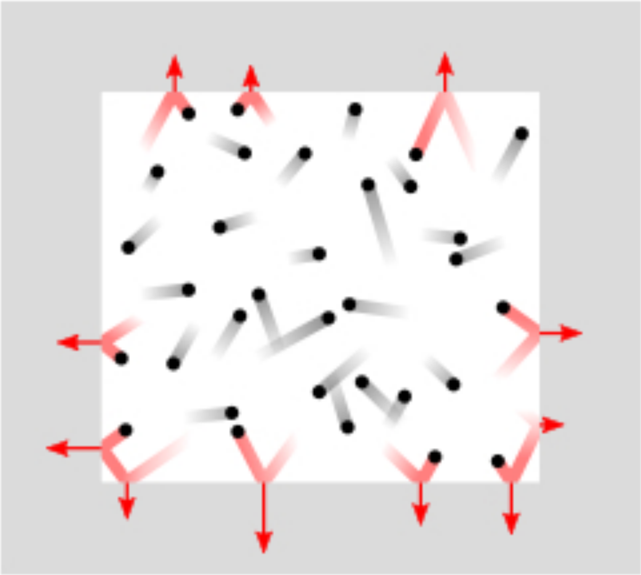
higher density *lower density*

specific gravity - ratio of an object's density to the density of water.

Pressure

$$P = \frac{F}{A}$$

P = pressure
F = force
A = area



$1\ N/m^2 = 1$ Pascal

1 atm = 101,000 Pa
1×10^5 Pa
exactly
1 bar

= 760 mm Hg

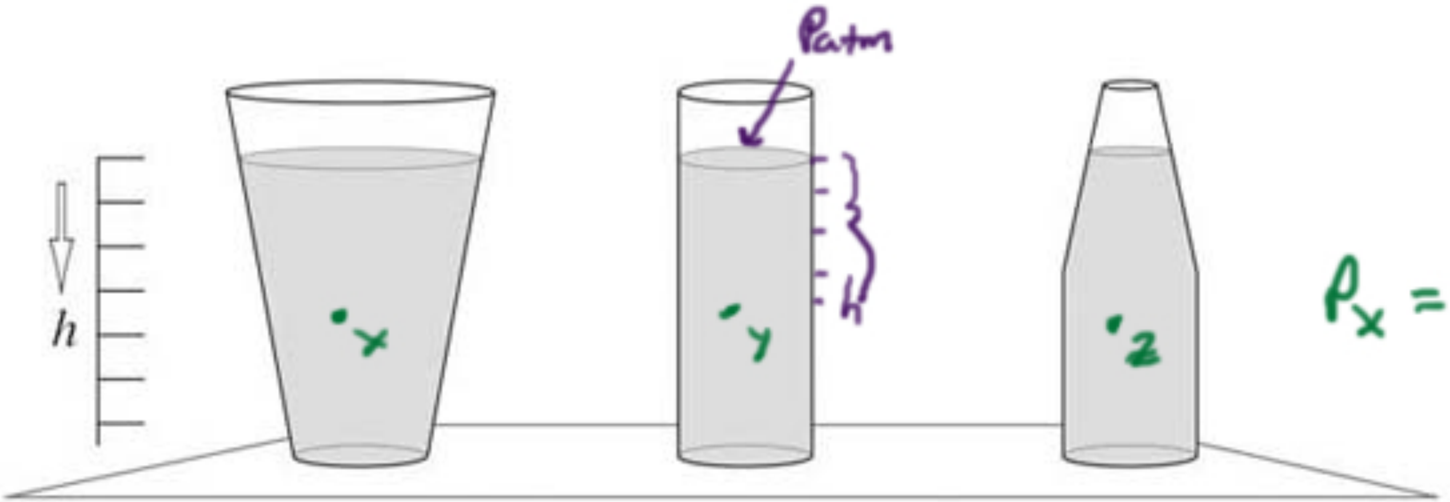
= 760 torr

Pressure Increases with Depth

$$P = P_a + \rho g h$$

density

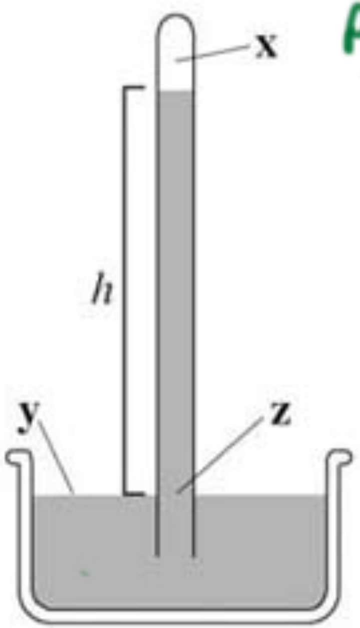
P = pressure
P_a = atmospheric pressure
ρ = density
g = acceleration due to gravity ($10 m/s^2$)
h = depth



$P_x = P_y = P_z$

Pressure is independent of the shape and size of the container.

With the common mercury barometer pictured at right, the atmospheric pressure is equal to the product of $\rho g h$ (ρ is the density of mercury). Which of the following is *not* always true with regard to this device when it is accurately measuring atmospheric pressure?



$P_x \sim$ vacuum

$P_z = P_y$

$P_y = P_{atm}$

$P_z = \rho_{Hg} g h$

$P_{atm} = \rho g h$

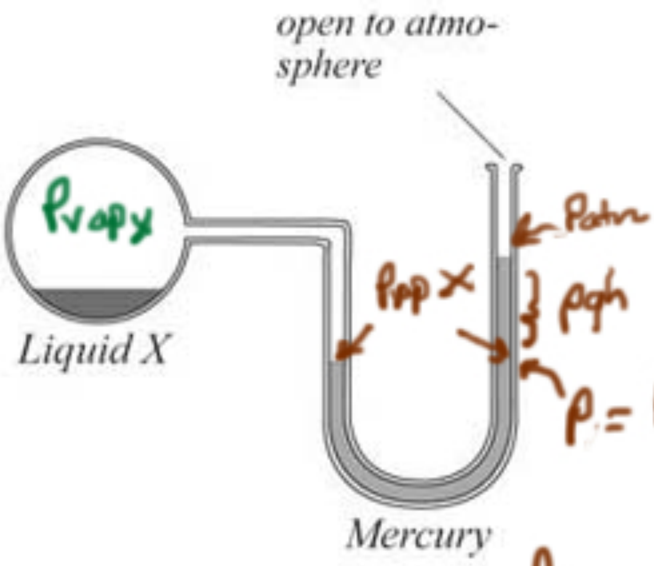
a. P_x is very nearly a vacuum

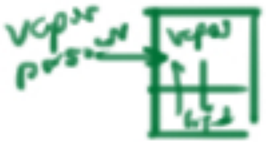
b. $P_y = P_z$

c. $P_y = P_{atm}$

d. $P_y = 760$ torr

After adding a 50 ml sample of *Liquid X* to the vacuum bulb at right, it was observed to boil within the bulb. After a time, the system reached the state shown at right, the boiling having ceased. Assume that the vapor pressure of mercury at this temperature is nearly zero (pressure of the gaseous phase above its liquid phase) what can we conclude from the experiment?





$P = P_{atm} + \rho g h$

$P_{vapX} = P_{atm} + \rho g h$

a. The density of *Liquid X* is greater than the density of mercury.

b. *Liquid X* possesses implausible properties.

c. The vapor pressure of *Liquid X* at room temperature is higher than atmospheric pressure.

d. The bouyancy of *Liquid X* is greater than the bouyancy of mercury.

Pascal's Law

$$\frac{F_a}{A_a} = \frac{F_b}{A_b}$$

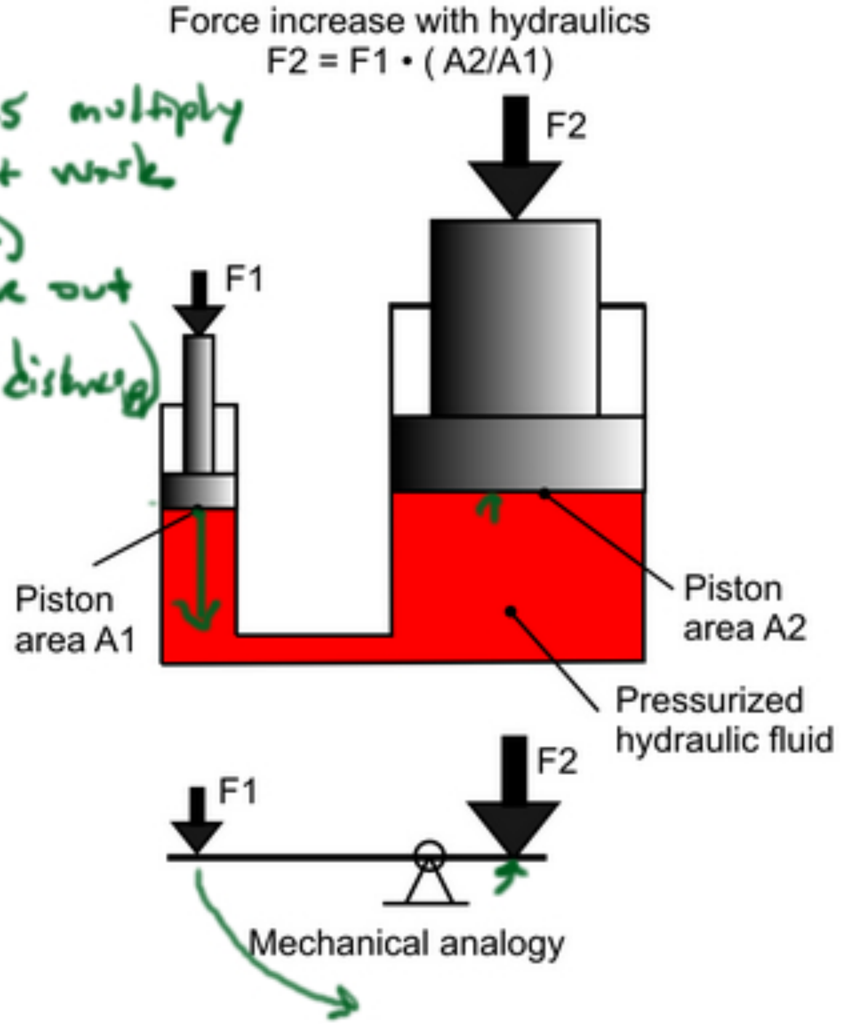
F = force
A = area

simple machines multiply force but not work
(at best)

work in = work out

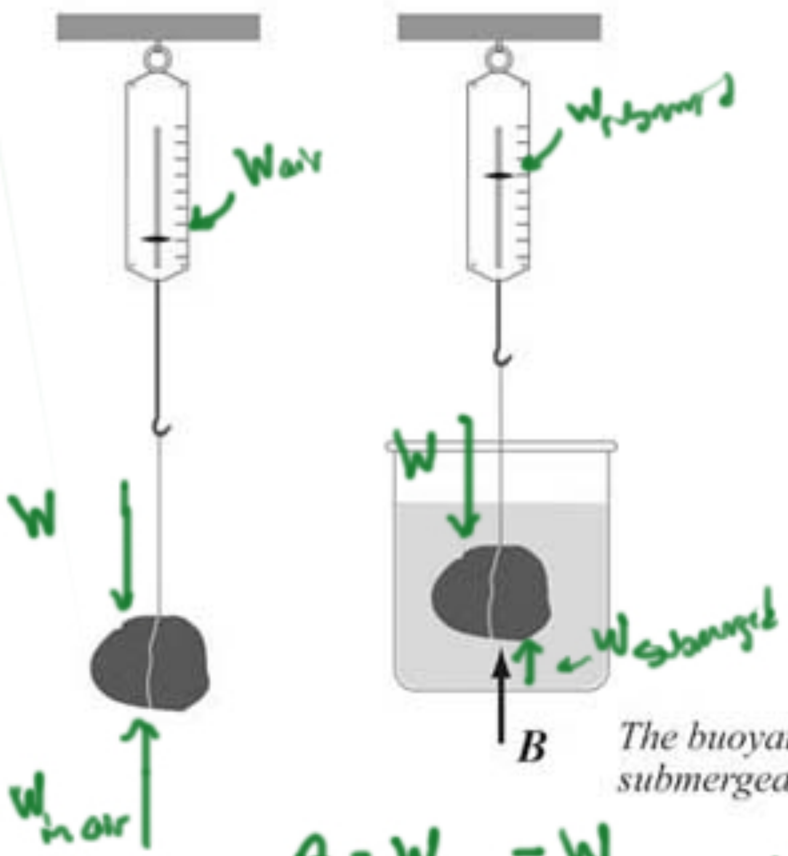
$(\text{force}_A)(\text{distance}_A) = (\text{force}_B)(\text{distance}_B)$

Pascal's Law states that an increase in the pressure on one of the surfaces enclosing a fluid will be transmitted as an undiminished increase in pressure to all parts of the fluid. This means for our hydraulic press at right that one hundred newtons exerting over one square meter, can hold up four hundred newtons over four square meters.



$$\text{specific gravity} = \frac{W_{air}}{W_{air} - W_{submerged}}$$

Archimedes' Principle



The buoyant force acting on a submerged mass

$$B = W_{air} - W_{submerged}$$

$W = W_{air}$

$$B = W_{\text{fluid displaced}}$$

B = Buoyant Force

$W_{\text{fluid displaced}}$ = weight of fluid displaced

- Submerged object -
  volume of object → volume of water
  → mass of water → weight = B

- floating object
  start with force equilibrium

% submerged
= specific gravity

A bar of lead has the dimensions $2 \times 3 \times 5\ cm^3$ and mass 330 g. What is the specific gravity of lead?

A. 1
B. 10
C. 11
D. cannot be determined from given information

ρ?

$\frac{330\,g}{30\,cm^3} = 11\ g/cm^3$

If the bar of lead were submerged in water, what would be the apparent loss of weight on the bar?

A. 300 N
B. 1 N
C. 0.3 N
D. 1/11 N

apparent loss of weight
means buoyant force.

displaces $30\ cm^3\ H_2O$
$= 30\ g\ H_2O = .03\ kg$

$W = mg = 0.3\ N$

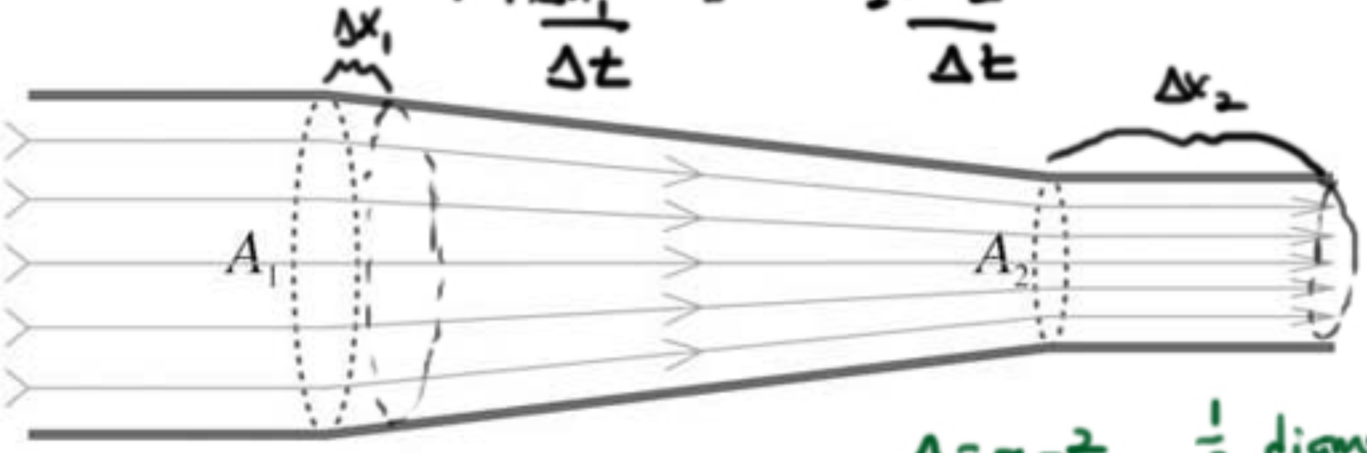
Continuity of Volume Flux

flow speed

$$A_1 v_1 = A_2 v_2 = \text{constant}$$

A = vessel cross-sectional area
v = flow speed

$$\frac{A_1 \Delta x_1}{\Delta t} = \frac{A_2 \Delta x_2}{\Delta t}$$



In the flow of an ideal fluid, the rate at which fluid volume moves through the vessel (volume flux) is the same everywhere along the pipe. Where vessel diameter is narrowest, the flow speed is greatest.

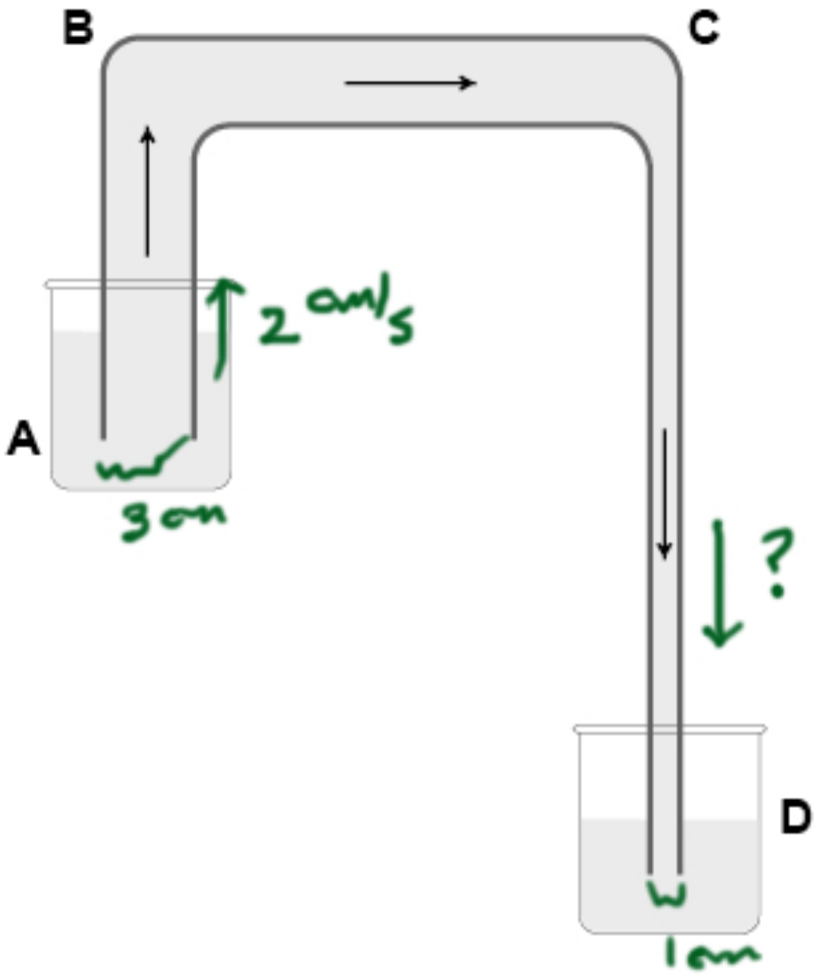
Ideal Fluid
- incompressible
- only laminar flow
- no viscosity

Volume Flux = $\frac{\text{volume}}{\text{time}}$

$\frac{\text{gal}}{s}$ $\quad$ m^3/s

Volume Flux = $Q = Av$

$A = \pi r^2$ $\quad$ $\frac{1}{2}$ diameter = 4 times flow speed



The diameter of tube segment AB is 3 cm. The diameter of tube segment CD is 1 cm. When the flow speed through AB is 2 cm/s, what will the flow speed be through CD?

A. 6 cm/s
B. 18 cm/s
C. 9 cm/s
D. 15 cm/s

$A_1 v_1 = A_2 v_2$

$A = \pi r^2$

$\left(\frac{1}{3} r\right)^2 = \frac{1}{9} r^2$

$\frac{1}{3}$ radius $\Rightarrow \frac{1}{9}$ area conservation of energy / volume element

Bernoulli's Equation

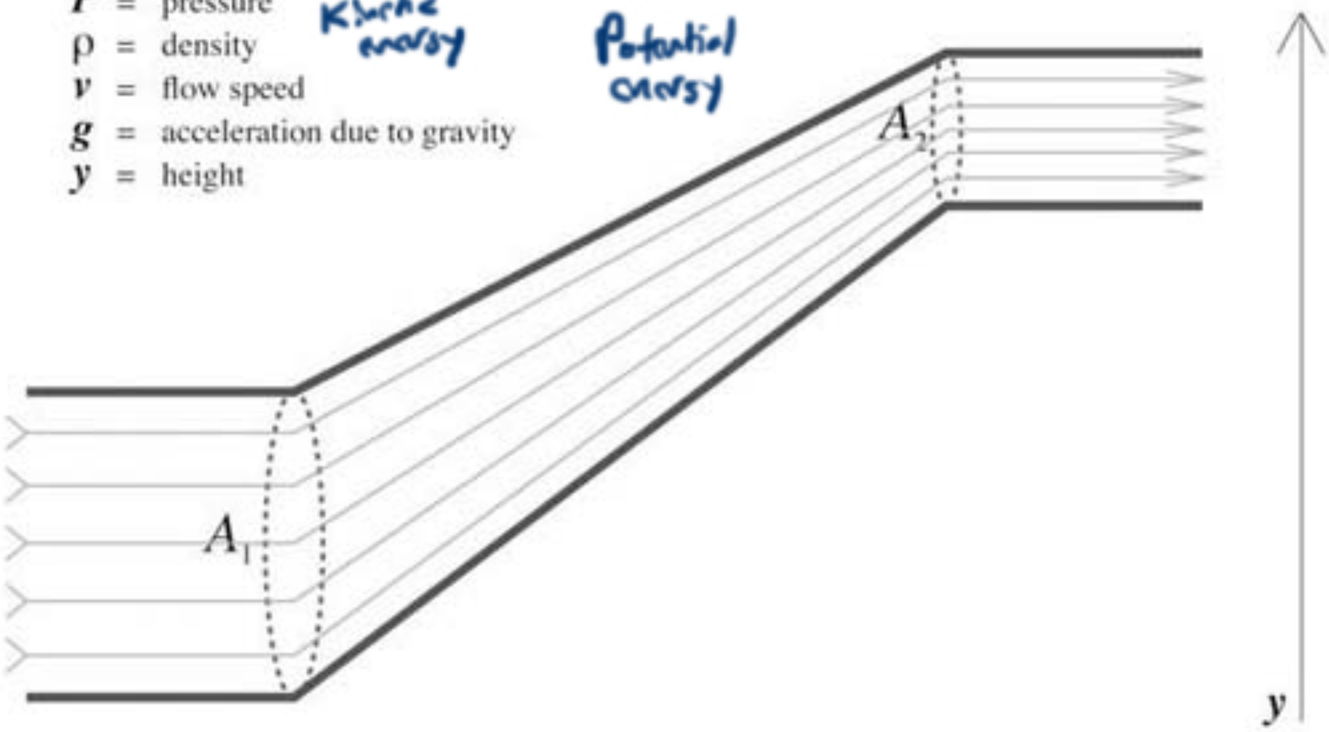
$$P + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}$$

pressure

Kinetic energy

Potential energy

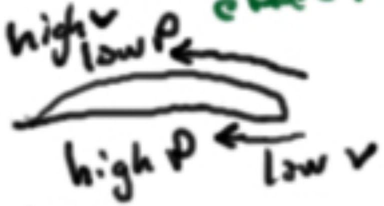
P = pressure
ρ = density
v = flow speed
g = acceleration due to gravity
y = height



By far the most common problem - level flow where interchange between P and $\frac{1}{2}\rho v^2$

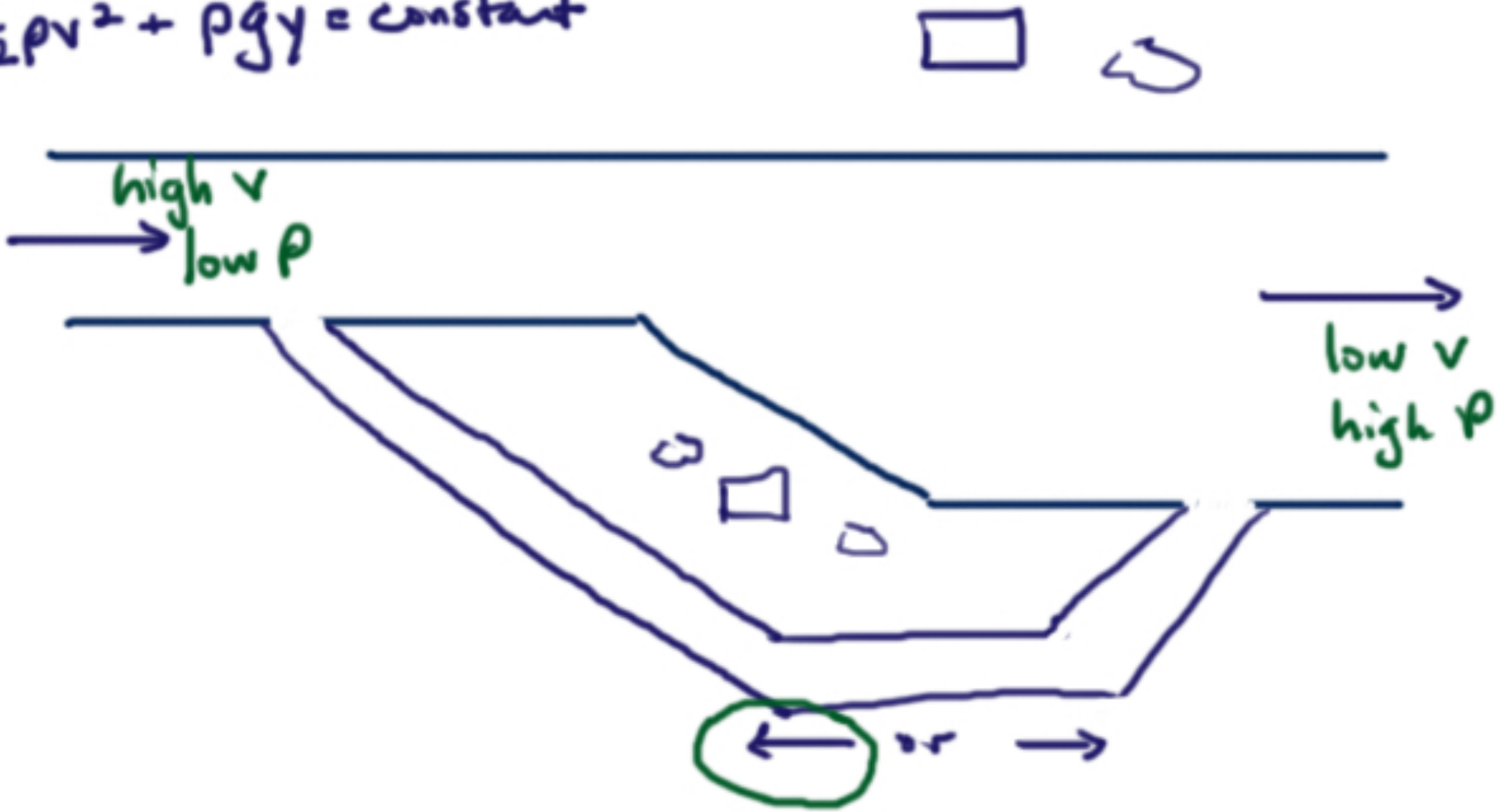
Where flow speed is high, pressure is low

↑ venturi effect



$$A_1 v_1 = A_2 v_2$$

$$p + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}$$



canals - uniform depth

bird's eye view

An aneurysm is caused by the weakening of the arterial wall where a bulge occurs and the cross-section of a vessel increases considerably. At the cross-section of an aneurysm

A. flow velocity will be reduced and the pressure will be reduced

B. flow velocity will be reduced and the pressure will increase

C. flow velocity will increase and the pressure will be reduced

D. flow velocity will increase and the pressure will increase

Real Fluids

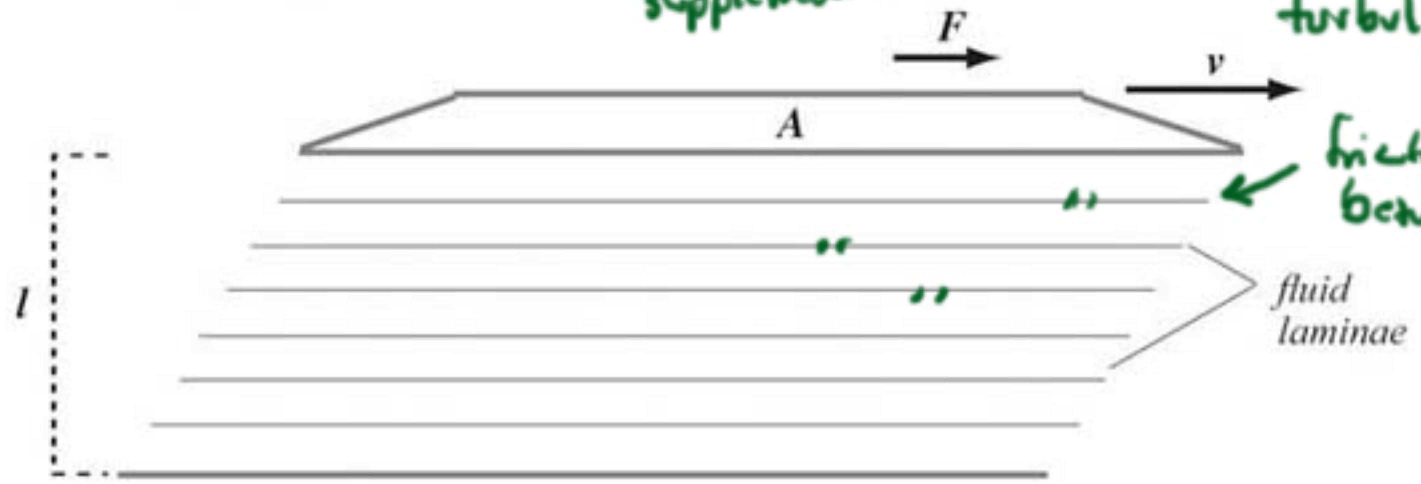
Viscosity

- η = viscosity
- F = shearing force
- A = sheared area
- v = sliding speed
- l = fluid thickness

$$\eta = \frac{F/A}{v/l}$$

supplemental

- Viscosity leads to resistance to flow (Poisseuille's Law)
- Viscosity opposes turbulence



friction between laminae

The figure above shows laminae, or layers of liquid, between two surfaces. The more viscous the fluid, the larger the shearing force required for the top surface to slide past the bottom surface at a given speed. Viscosity reflects friction between the laminae, or fluid layers.

The Causes of Turbulent Flow

density ↘ speed ↙ ← geometric property

$$RN = \frac{\rho v d}{\eta}$$

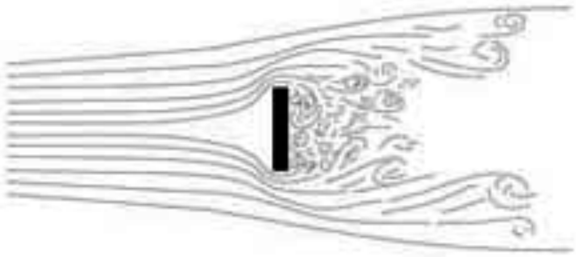
↑ viscosity

RN = Reynolds number
ρ = fluid densitiy
v = flow speed
d = geometrical property of the flow (diameter of obstruction, pipe width)
η = viscosity

$RN > 3000$ turbulance is likely

laminar (streamline) flow

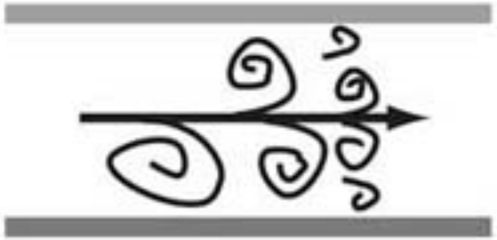
(low Reynolds number)



turbulent flow

(high Reynolds number)

Most blood flow is remarkably free of turbulence, although, under both normal or abnormal conditions, turbulence may occur in certain areas of the circulatory system. Which of the following would be most likely to directly contribute to turbulence?



$$RN = \frac{\rho v d}{\eta}$$

$$A_1 v_1 = A_2 v_2 = Q$$

a. ~~decreased cardiac output~~
b. ~~increased blood viscosity~~
c. localized narrowing of an arterial vessel
d. decrease in blood density

!

Poiseuille's Law

2 ways to look at this

$$Q = \frac{\Delta P \, \pi \, r^4}{8 \, \eta \, l}$$

Q	=	volume flux
ΔP	=	change in pressure
r	=	pipe or vessel radius
η	=	viscosity
l	=	pipe or vessel length

ΔP as effect

1) What leads to high Q (volume flux)

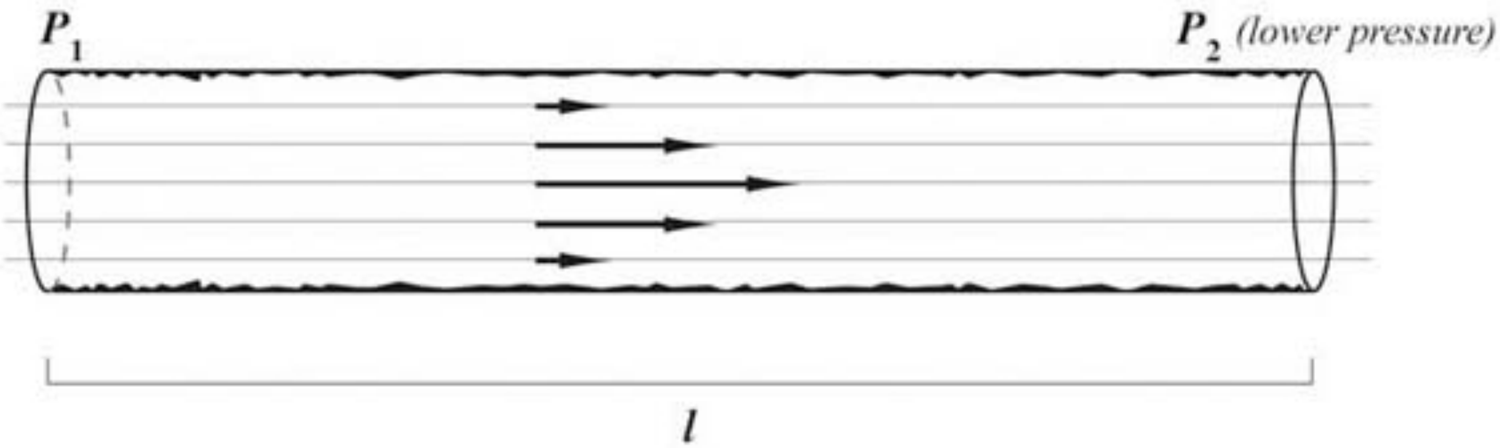
$\frac{\Delta P}{l}$ big pressure gradient $\quad r^4$ large vessel $\quad \eta$ low viscosity

2) What leads to a big ΔP?

Viscous dissipation

small r, big η

ΔP as cause



$$Q = \frac{\Delta P \pi r^4}{8 \eta l}$$

With the plunger exerting the same negative pressure, the total time required to draw 10ml of blood through a syringe with a needle having half the length and half the diameter would be

A. 1/2 as long
B. the same
C. 8 times longer
D. 32 times longer

$\frac{1}{2} l \Rightarrow 2 \times Q$

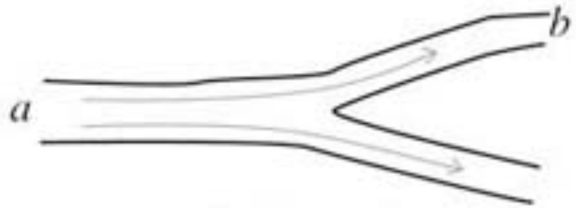
$\frac{1}{2} r \Rightarrow \frac{1}{16} \times Q$

$\frac{1}{8} Q$

A large artery branches into two smaller arteries, each with 50% of the radius of the larger. Since all blood flowing into the junction enters the smaller arteries, the volume flux within the larger must be twice the volume flux through either of the smaller. Thus, from Poiseuille's Law:

$$\frac{\Delta P_a r_a^4}{l_a} = \frac{2\Delta P_b r_b^4}{l_b}$$

Which follows from this relationship?



$$Q = \frac{\Delta P \pi r^4}{8 \eta l}$$

Poiseuille's Law

Q = volume flux
ΔP = change in pressure
r = vessel radius
η = viscosity
l = vessel length

$r_b = \frac{1}{2} r_a$

$\frac{\Delta P_a}{l_a} = \frac{1}{8} \frac{\Delta P_b}{l_b}$

$\frac{\Delta P}{l}$

a. Pressure drop per unit length is much greater in a small artery than in the large.

b. Pressure drop per unit length is double in the large artery vs. the small.

c. Total flow speed in the small arteries equals total flow speed in the large.

d. Pressure drop per unit length is double in the small artery vs. the large.

The answer is a

Because of the extreme effect of the decreasing radius on viscous dissipation as described by Poiseuille's Law, decreasing the radius causes a very large increase in the pressure drop per unit length.

In our example, if we substitute $(0.5)r_a$ for r_b:

$$\frac{\Delta P_a r_a^4}{l_a} = \frac{2\Delta P_b r_b^4}{l_b}$$

$$\frac{\Delta P_a r_a^4}{l_a} = \frac{2\Delta P_b (0.5\ r_a)^4}{l_b}$$

$$\frac{\Delta P_a}{l_a} = \frac{1}{8}\frac{\Delta P_b}{l_b}$$

A MAJOR CONSEQUENCE OF POISEUILLE'S LAW IS THAT IN THE CARDIOVASCULAR SYSTEM, THE NARROW VESSELS ARE MUCH SHORTER. IF NOT, THERE WOULDN'T BE PRESSURE REMAINING AFTER BLOOD FLOWS THROUGH THE CAPILLARIES FOR IT TO RETURN TO THE HEART.