Ideal Gas & Kinetic Theory Practice Items

1. Which of the following volumes is closest to that occupied by one mole of an ideal gas at standard state temperature and standard pressure?

- **A.** 1 liter
- **B.** 18.9 liters
- **C.** 22.4 liters
- **D.** 24.4 liters

2. A constant volume gas thermometer will be most accurate

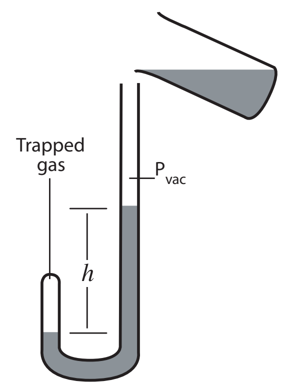
- **A.** at low pressures
- **B.** at low temperatures
- **C.** when conditions for the gas are near the condensation stage
- **D.** if the gas is high density

3. In the interstellar regions of the Milky Way, a temperature of 2.7 K arises from radiation and particle kinetic energy. Matter exists in the form of approximately one million neutral hydrogen atoms per cubic meter. Which is the best estimate of the pressure in interstellar space?

- **A.** 3.5×10^{-19} Pa
- **B.** 3.6×10^{-17} Pa
- **C.** 2.2×10^{-7} Pa
- **D.** 2.7×10^{-6} Pa

4. The U-tube pictured below contains a volume, V_g, of ideal gas trapped by a column of mercury under a vacuum. As mercury is added to the U-tube with the system maintained at constant temperature,

- **A.** the product $V_g h$ maintains a constant value as h increases.
- **B.** the product $V_g h$ decreases.
- **B.** the product $V_g h$ increases.
- **C.** heat flow occurs into the gas.



5. Which of these gases has the lowest molar heat capacity?

- **A.** Ar
- **B.** H_2
- **C.** NH_3
- **D.** C_3H_8

6. What is the approximate ratio of the specific heat (cal/g, constant volume) of neon gas (MW 20.2 g) to the specific heat of krypton gas (MW 83.8 g)?

- **A.** 1:4
- **B.** 1:1
- **C.** 4:1
- **D.** not enough information to determine

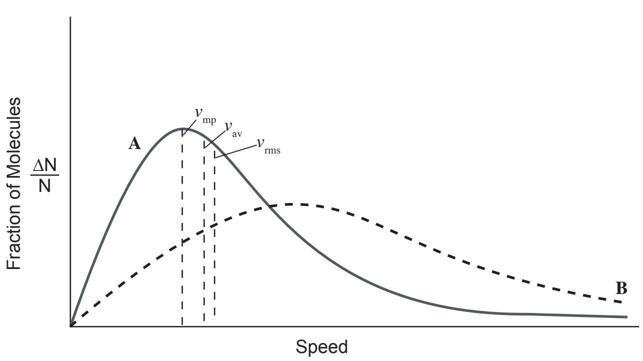
7. Almost all atoms in a sample of helium gas become completely ionized when the thermal energy reaches approximately 3.5×10^3 kJ/mol. At what approximate temperature does helium gas completely ionize?

A. 3,500 K
B. 5,250 K
C. 10,500 K
D. 28,000 K

Passage (Questions 8-12)

Gas molecules move rapidly in all directions in a random fashion. For a given temperature, the Maxwell-Boltzmann distribution describes the variation of speeds among the molecules. The distribution is often represented graphically as a plot of the fraction of molecules vs. speeds. The fraction of molecules with very high speeds or very low speeds is small. The peak of the graph corresponds to the most probable speed. A somewhat higher speed than the most probable speed is the root mean square speed, RMS, which corresponds to the speed of a particle with average kinetic energy. The average speed itself is $0.913 \times$ RMS.

The graph below illustrates the Maxwell-Boltzmann distributions for two gases of equal molecular size present at equimolar concentrations in two separate stoppered flasks. The distribution for gas A is shown with the solid line. The distribution for gas B is shown with the dotted line.



8. In comparison to gas A, Gas B

A. is at a higher temperature.
B. is at a lower temperature.
C. has larger molecules.
D. has smaller molecules.

9. In a sample of gas more molecules have this particular speed than any other.

A. v_{mp}
B. v_{av}
C. v_{rms}
C. impossible to determine

10. Assuming the temperature of gas A were 300K, what is the approximate temperature of gas B?

A. 150 K
B. 425 K
C. 600 K
D. 1200K

11. A graph illustrating the Maxwell-Boltzmann distributions of which of the following two gases at equimolar concentrations in thermal equilibrium would most closely resemble the graph in the passage?

A. O_2 and N_2
B. Ar and Ne
C. CH_4 and He
D. NH_3 and H_2

12. In kinetic theory the mean free path of a particle is the average distance the particle travels between collisions with other moving particles. For gas particles it may be shown that the mean free path, in meters, is

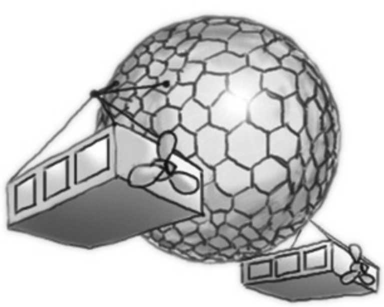
$$l = \frac{k_b T}{\sqrt{2}\pi d^2 P}$$

where k_b is the Boltzmann constant in J/K, T is the temperature in K, P is pressure in Pascals, and d is the diameter of the gas particles in meters.

Which of the following would cause an increase in the mean free path of the particles of a gas?

A. increasing the temperature of the gas at constant volume
B. increasing the pressure of the gas at constant volume
C. increasing the volume of the gas at constant temperature
D. ionizing the gas with an electron beam

Passage (Questions 13-16)



A large vacuum dirigible, such as depicted in the artist's rendering above, would produce enormous lift. A spherical vacuum dirigible constructed of a geodesic sphere 200m in diameter with 1.2×10^6 kg devoted to its aluminum frame and carbon fiber skin with 0.5atm internal pressure would be capable of floating in the Earth's atmosphere carrying approximately 1.5×10^6 kg in equipment, cargo and passengers in addition to the mass of its structure and cladding.

Current techniques employed in the design and construction of geodesic domes could produce an extremely stable geodesic sphere as described, prior to evacuation. However, the true challenge would be to construct an airship that could withstand the compressive forces after the vacuum has been introduced. The difficulty of the engineering challenges involved can be appreciated in the determination of the material constraints for a vacuum dirigible comprised of a homogeneous spherical shell enclosing a total vacuum. The total force on a spherical shell of radius R by an external pressure P is $\pi R^2 P$. The force on each hemisphere in equilibrium along the equator will produce the compressive stress given below:

$$\sigma = \frac{\pi R^2 P}{2\pi R h} = \frac{RP}{2h}$$

where h is the shell thickness.

Neutral buoyancy occurs when the shell has the same mass as the displaced air, which occurs when $h/R = \rho_a/(3\rho_s)$, where ρ_a is the air density and ρ_s is the shell density. Combining with the stress equation gives

$$\sigma = \frac{3\rho_s P}{2\rho_a}$$

For terrestrial conditions such a degree of stress is of the same order of magnitude as the compressive strength of aluminum alloys, arguing for the feasibility of the spherical shell design.

Unfortunately this disregards buckling. Using the formula for the critical buckling pressure of a sphere

$$P_{cr} = \frac{2Eh^2}{\sqrt{3(1-\mu^2)}} \frac{1}{R^2}$$

where E is the modulus of elasticity and μ is the Poisson ratio of the shell material, ie. the relationship of transverse bulging to axial compression. (Most potential shell materials possess a Poisson ratio of approximately 0.3). Substituting the condition for neutral buoyancy, $h/R = \rho_a/(3\rho_s)$, gives a necessary condition for a feasible vacuum balloon shell:

$$\frac{E}{\rho_s^{\,2}} = \frac{9P_{cr}\sqrt{3(1-\mu^2)}}{2\rho_a}$$

The requirement is about 4.5×10^5 $kg^{-1}m^5s^{-2}$. This cannot even be achieved using diamond ($E/\rho_s^{\,2} \approx 1.0 \times 10^5$ $kg^{-1}m^5s^{-2}$).

In summary, a vacuum dirigible comprised of a homogeneous spherical shell does not appear to be possible given currently available materials. However, dropping the assumption that the shell is a homogeneous material may allow lighter and stiffer structures such as with a honeycomb structure or geodesic construction. A number of engineering groups have claimed success in creating viable designs in recent years although there have been no public demonstrations of a working prototype.

13. A vacuum dirigible could lose buoyancy through the influx of gas molecules through pinhole defects in its carbon fiber skin. Assuming the airship material and air density remained unchanged as temperature increased, the increase in the rate of gas influx in moving from a 0°C environment to a 30°C environment would be approximately

A. 3%
B. 10%
C. 17%
D. 81%

14. A hypothetical vacuum airship achieves neutral buoyancy with an interior pressure of 0.5 atm. What approximate interior temperature within 27°C surroundings would a hot-air balloon of the same volume and mass need to establish to generate the same lift?

A. 54°C
B. 129°C
C. 254°C
D. 327°C

15. According to the passage, construction of a viable vacuum airship comprised of a homogeneous spherical shell might be constructed if a material with the following properties compared to diamond could be developed:

A. 1/2 the rigidity and 1/4 the density
B. the same rigidity and 1/2 the density
C. twice the rigidity with the same density
D. twice the rigidity and twice the density

16. A vacuum dirigible constructed of a homogeneous spherical shell is to be developed for flight within a dense low pressure extra-terrestrial atmosphere. From the information presented in the passage it can be deduced that increasing the design radius

A. would require an increase in the thickness of the shell material directly proportional to the increased radius.
B. would lead to more favorable material constraints as radius increased.
C. would enable the dirigible to operate in higher temperatures.
D. would make it more possible for the dirigible to be constructed from a material having a low Poisson ratio.

Ideal Gas & Kinetic Theory
Answers and Explanations

1. D
One mole of an ideal gas occupies 22.4 L at STP. This fact you absolutely must memorize for the MCAT. The temperature at STP is 273 K. This is standard temperature. However, 273 K is not *standard state temperature*. Standard state temperature is not 273 K but 298K. This may seem ridiculous, but there is a figure of merit to knowing about this difference. Many bench-top measurements are at 298 K not 273 K. Charles' Law tells us that at constant pressure the volume of an ideal gas sample is directly proportional to its temperature, i.e. V/T = constant, so at 298 K the volume will be greater than 22.4 L.

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\frac{22.4\text{ L}}{273\text{ K}} = \frac{24.4\text{ L}}{298\text{ K}}$$

2. A
This question is really asking 'when will a real gas behave most like an ideal gas?' Generally, a gas behaves more like an ideal gas at higher temperature and lower pressure, as the potential energy due to intermolecular forces becomes less significant compared with the particles' kinetic energy, and the size of the molecules becomes less significant compared to the empty space between them.

3. B
To use the ideal gas law, first we need to know how many moles is represented by one million atoms.

$$1 \times 10^6 \text{ particles} \left(\frac{1 \text{ mole}}{6.02 \times 10^{23} \text{ particles}} \right)$$

$$= 0.16 \times 10^{-17} = 1.6 \times 10^{-18} \text{ mole}$$

Now we can determine the pressure. Because the answer choices are spaced numerically, you have plenty of latitude for mental math with these computations.

$$PV = nRT$$

$$P = \frac{nRT}{V}$$

$$P = \frac{(1.6 \times 10^{-18}\text{ mole})(8.3\text{ J mole}^{-1}\text{ K}^{-1})(2.7\text{ K})}{1\text{ m}}$$

$$P = \frac{(1.6 \times 10^{-18}\text{ mole})(8.3\text{ J mole}^{-1}\text{ K}^{-1})(2.7\text{ K})}{1\text{ m}}$$

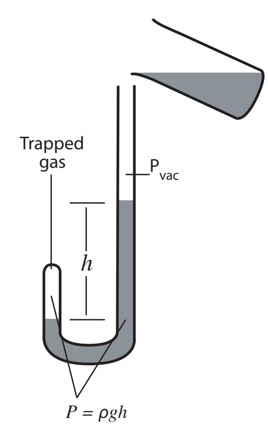
$$P = 3.6 \times 10^{-17}\text{ Pa}$$

4. A
Boyle's Law:

$$P_1 V_1 = P_2 V_2 \quad \text{constant temperature}$$

At constant temperature the pressure of an ideal gas sample is inversely proportional to its volume, i.e. PV = constant.

The pressure in our trapped gas is directly proportional to the height of the mercury column.



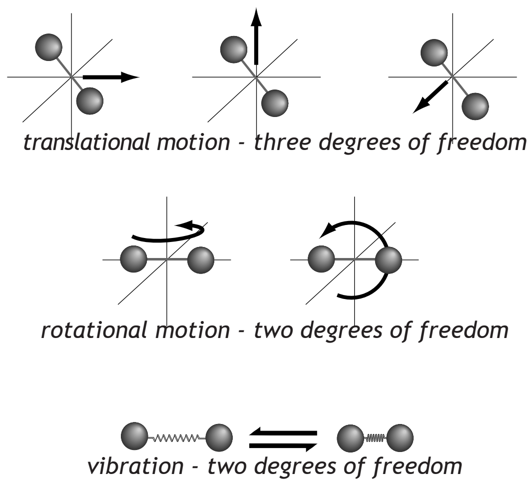
$$P_g V_g = \text{constant} \qquad P_g = \rho gh$$

$$\rho gh V_g = \text{constant}$$

$$hV_g = \text{constant}$$

5. A

A monatomic gas molecule such as argon possesses only kinetic energy deriving from its linear motion. A diatomic gas molecule, like H_2, in addition to translational motion, can also rotate and vibrate.



With the ability to store energy in both vibrational and rotational modes, a diatomic gas has more partitions for thermal energy. As a sample of diatomic gas takes in heat, the energy spreads out into all of its degrees of freedom. The other choices besides argon can absorb heat flow into all modes, translational, rotational, and vibrational. For this reason, the molar heat capacity of the other gaseous substances is greater than the molar heat capacity of argon.

6. C

Because neon and krypton both exist as monatomic gases, being noble gases, they both possess kinetic energy at the particle level, ie. thermal energy, only in the form of translational kinetic energy. There are no rotational or vibrational modes. If two substances have the same number of places to put kinetic energy at the particle level, they will have the same molar heat capacity. In the case of neon and krypton, their respective constant volume molar heat capacities will be equal and very close to the ideal gas value.

$$C_V = \frac{3}{2}R = 12.5\ \frac{\text{J}}{\text{mol K}}$$

If their molar heat capacities are very close (joules per mole degree Kelvin), then their specific heats will be different (joules per gram degree Kelvin) according to molecular weight.

$$c_{\text{neon}} = \left(\frac{12.5\ \text{J}}{\text{mol K}}\right)\left(\frac{\text{mol}}{20.2\ \text{g}}\right)$$

$$c_{\text{kryp}} = \left(\frac{12.5\ \text{J}}{\text{mol K}}\right)\left(\frac{\text{mol}}{83.8\ \text{g}}\right)$$

$$c_{\text{neon}} : c_{\text{kryp}} \approx 4 : 1$$

7. D

We can apply the ideal gas model. Helium, a monatomic noble gas, is very near to ideal gas behavior. In a sample of an ideal gas, internal energy is only in the form of thermal energy. The internal energy of an ideal gas is directly proportional the temperature.

$$U = \tfrac{3}{2}nRT$$

$$3.5 \times 10^6\ \text{J} = \tfrac{3}{2}\ (1\ \text{mol})\ (8.3\ \frac{\text{J}}{\text{mol K}})\ T$$

$$T = 2.8 \times 10^5\ \text{K}$$

8. A

Gas B is at a higher temperature. As the temperature of the molecules represented by a Maxwell-Boltzmann distribution increases, the distribution flattens out. rms speed is higher. This corresponds to a greater average kinetic energy per particle, a higher temperature.

9. A

The speed at the top of the curve is called the most probable speed because the largest number of molecules have that speed.

10. D

The temperature of ideal gas is directly proportional to the translational kinetic energy of the particles. Real gases may have thermal energy in the form of rotational and vibrational kinetic energies as well, so with real gases it's more proper to say that the relationship is between temperature and kinetic energy per degree of freedom within translational, rotational, and vibrational modes. Nevertheless, whether the gas is ideal or real, temperature increases with the average translational kinetic energy per particle.

$$\tfrac{1}{2}m\overline{v^2} = \tfrac{3}{2}kT$$

The particles in gas B are moving at approximately twice the rms speed, so the average kinetic energy per particle is four times greater and so is the temperature.

11. C

For this question we assume the distribution represents two gases in thermal equilibrium. In other words, they are the same temperature. At the same temperature, the average translational kinetic energy per particle will be the same, yet the particles of gas B are moving twice as fast on average.

$$\tfrac{1}{2}m_A\overline{v_A^2} = \tfrac{1}{2}m_B\overline{v_B^2}$$

If gas B particles are moving twice as fast, to possess the same translational kinetic energy, the mass of gas B particles must be ¼ the mass of gas A particles. The only pair satisfying this condition are CH_4 (MW 16u) and He (MW 4u).

12. C

Neither 'A' nor 'B' is correct because temperature and pressure go up or down together in direct proportionality at constant volume. Both choices 'A' and 'B' would produce no change in the mean free path. Choice 'C' however would increase the mean free path because an increase in volume at constant temperature would lead to a decrease in pressure.

13. A

Comparing gases at the same temperature, Graham found experimentally that the rate of effusion of a gas is inversely proportional to the square root of the masses of the particles. This is because effusion rate is proportional to the rms speed of the particles. In this problem, the temperature of the gas increased. A difference of 0°C and 30°C is a difference of 273K and 303K. The Kelvin temperature increased approximately 10%. The average translational kinetic energy of the gas particles is directly proportional to the Kelvin temperature.

$$\tfrac{1}{2}m\overline{v^2} = \tfrac{3}{2}kT$$

A 10% increase in Kelvin temperature corresponds to a 10% increase in translational kinetic energy, which, in turn, corresponds to approximately a 3% increase in rms speed (√10).

14. D

For the buoyant force to be equal and opposite to the weight of the aircraft in both cases (neutral buoyancy), the air inside the hot-air balloon would need to weigh the same as the air inside the vacuum dirigible at 0.5 atm. In other words, it needs to be half as dense as the surrounding air. In a hot air balloon, the air within the balloon is the same pressure as the surrounding air. To possess the same pressure and half the density, the Kelvin temperature needs to be twice as great. You can see this in the ideal gas law.

$$PV = nRT$$

$$\frac{n}{V} = \frac{P}{TR}$$

We are given a temperature for the surroundings of 27°C, which equals 300K ($T = T_c + 273.15$). Doubling 300K to 600K and converting back to Celsius gives us 327°C.

15. A

This passage has a number of difficult out-of-scope elements. It's important to remember in the exam that a passage like this isn't about foreknowledge of the out-of-scope elements. Ultimately, the questions are going to be about fundamentals and how well you kept your footing. An important figure of merit when you see an unfamiliar equation is to get yourself on speaking terms with it. What is the equation saying? What changes with what?

An elastic modulus indicates how difficult a material is to deform under stress. A material possessing a high elastic modulus has a high rigidity. In the passage we are told that the ratio of elastic modulus to the square of density of diamond is $E/\rho_s^2 \approx 1.0 \times 10^5$ $kg^{-1}m^5s^{-2}$, but the minimum requirement to prevent buckling for a vacuum dirigible of neutral buoyancy is presented as $E/\rho_s^2 = 4.5 \times 10^5$ $kg^{-1}m^5s^{-2}$. If a material were developed where the elastic modulus were halved while also quartering density that would increase E/ρ_s^2 eight-fold, so while this material would be less rigid than diamond its lower density (and thus weight) would reduce the stresses it would need to undergo and thus make a viable vacuum dirigible possible.

16. A

Neutral buoyancy determines a ratio of shell thickness to radius as a function of the ratio of the density of the surrounding air to the density of the shell material.

$$h/R = \rho_a/(3\rho_s)$$

As the radius increases the shell thickness must increase in direct proportion.